Enrollment No: ______________________ Exam Seat No:______________________

C.U.SHAH UNIVERSITY

Summer Examination-2017

Subject Name: Advanced Complex Analysis

Subject Code: 5SC04CAC1 **Branch: M.Sc (Mathematics)**

Semester: 4 **Date 12/04/2017** **Time : 10:30 To 01:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions.** **(07)**

a. Define zero of function of multiplicity m. **(02)**

b. Evaluate $\oint_C \frac{z}{z^2+9}$ where C: $|z|$=5. **(02)**

c. State open mapping theorem. **(02)**

d. Define: Smooth curve. **(01)**

Q-2 **Attempt all questions** **(14)**

a. State and prove Morera's theorem. **(07)**

b. Letγ:[0,1]→ Ċ be closed rectifiable curve and a $\notin \{\gamma\}$then show that $\frac{1}{2\pi i}\int_\gamma \frac{dz}{z-a}$ is an integer. **(07)**

OR

Q-2 **Attempt all questions** **(14)**

a. State and prove Cauchy's integral formula first version. **(07)**

b. Let γ be a rectifiable curve. Suppose $\emptyset$ is a complex valued function defined on $\{\gamma\}$. If $f_m(z) = \int_\gamma \frac{\emptyset(w)dw}{(w-z)^m}\, dw, z \notin \{\gamma\}$ for $m \geq 1$ then prove that f_m's are analytic on $G = \mathbb{C} - \{\gamma\}$ and $f_m{}'(z) = m\, f_{m+1}(z)$.. **(07)**

Q-3 **Attempt all questions** **(14)**

a. Show that zero of an analytic function is always isolated. **(06)**

b. State and prove counting zero principle. **(05)**

c. Evaluate $\frac{1}{2\pi i}\int_{|z|=3} \frac{2z+1}{z^2+z+1}\,dz$ **(03)**

OR

Q-3

a. State and prove argument principle. **(07)**

b. Evaluate $\frac{1}{2\pi i}\int_{\gamma} \frac{f'(z)}{f(z)}\,dz$ Where $f(z)=(z^2+2)^3$ and $\gamma(t) = 3e^{it}$, $0 \le t \le 2\pi$. **(05)**

c. Define: Meromorphic function. **(02)**

SECTION – II

Q-4 **Attempt the Following questions.** **(07)**

a. Define convex set function. **(02)**

b. State Montel's theorem. **(02)**

c. Define:Conformal map **(02)**

d. Define: Removable singularity. **(01)**

Q-5 State and prove Rouche's theorem also deduce fundamental theorem of algebra from it. **(14)**

OR

Q-5 **Attempt all questions** **(14)**

a. Prove that a differentiable function f is convex iff f' is increasing. **(07)**

b. Prove that $f: [a, b] \to R$ is convex function if and only if the set $A=\{ (x, y) \in R^2 / f(x) \le y\}$ is a convex set. **(04)**

c. State Hadamard's three circles theorem. **(03)**

Q-6 **Attempt all Questions** **(14)**

a. State and prove Luca's theorem **(07)**

b. Prove that a function f is convex on (a,b) if and only if for each triplet points $s, t, u$with $s \le t \le u \frac{f(t)-f(s)}{t-s} \le \frac{f(u)-f(t)}{u-t}$, $a < s < t < u < b$. **(07)**

OR

Q-6 **Attempt all Questions**

a. State and prove Weiestrass factorization theorem. **(07)**

b. State and prove Jensen's formula. **(07)**